Inverse of matrix

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Finding the Inverse of a 2x2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Step-1 First find what is called the Determinant

This is calculated as ad-bc

Step-2 Then swap the elements in the leading diagonal

$$\begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

Step-3 Then negate the other elements

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Step-4 Then multiply the Matrix by 1/determinant

$$\frac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example Find Inverse of A

Step 1 – Calc Determinant

$$A = \begin{pmatrix} 4 & 8 \\ 1 & 3 \end{pmatrix}$$

Determinant (ad-cb) = 4x3-8x1 = 4

Step 2 – Swap Elements on leading diagonal

step2 $\begin{pmatrix} 3 & 8 \\ 1 & 4 \end{pmatrix}$

Step 3 – negate the other elements

step3 $\begin{pmatrix} 3 & -8 \\ -1 & 4 \end{pmatrix}$

Step 4 – multiply by 1/determinant

step4 $\frac{1}{4}\begin{pmatrix} 3 & -8 \\ -1 & 4 \end{pmatrix}$

$$A^{-1} = \begin{bmatrix} 0.75 & -2 \\ -0.25 & 1 \end{bmatrix}$$

check

$$AA^{-1} = \begin{bmatrix} 4 & 8 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0.75 & -2 \\ -0.25 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & -8+8 \\ 0.75-0.75 & -2+3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find the inverses and check them

$$A = \begin{pmatrix} 2 & 6 \\ 1 & 5 \end{pmatrix} \qquad A^{-1} = \frac{1}{4}\begin{pmatrix} 5 & -6 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1.25 & -1.5 \\ -0.25 & 0.5 \end{pmatrix}$$

$$B = \begin{pmatrix} -5 & 20 \\ -1 & 2 \end{pmatrix} \qquad B^{-1} = \frac{1}{10}\begin{pmatrix} 2 & -20 \\ 1 & -5 \end{pmatrix} = \begin{pmatrix} 0.2 & -2 \\ 0.1 & -0.5 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix} \qquad C^{-1} = \frac{1}{-2}\begin{pmatrix} -1 & -2 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0.5 & 1 \\ 0 & -1 \end{pmatrix}$$

We can use A^{-1} to solve a system of equations

$$x + 3y = 1$$

$$2x + 5y = 3$$

To see how, we can re-write a system of equations as matrices.

$$A\mathbf{x} = \mathbf{b}$$

coefficient matrix

variable matrix

constant matrix

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Why will it help us solve equations?

Because if we can express a system of equations in the form

$$Ax = b$$

Then we can multiply both sides by the inverse matrix

$$A^{-1}Ax = A^{-1}b$$

And we can then know the values of X because $A^{-1}A = I$

$$\boldsymbol{x = A^{-1}b}$$

THANK YOU