CHARACTERISTIC VECTORS OF EIGEN VECTORS

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CHARACTERISTICS VECTORS OR EIGEN VECTORS

A column vector *X* is transformed into column vector *Y* by means of a square matrix *A*.

Now we want to multiply the column vector *X* by a scalar quantity **λ** so that we can find the same transformed column vector *Y*.

i. e., $AX = \lambda X$

X is known as eigen vector.

Example 1) Show that the vector (1,1,2) is an eigen vector of the matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

corresponding to the eigen value 2.

Solution . Let $X=(1,1,2)$.

$$\text{Now, } AX = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+1-2 \\ 2+2-2 \\ 2+2+0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 2X$$

Corresponding to each characteristic root λ, we have a corresponding non-zero vector *X* which satisfies the equation $[A-\lambda I]X=0$. The non-zero vector *X* is called characteristic vector or Eigen vector.

PROPERTIES OF EIGEN VECTORS-

1)The eigen vector *X* of a matrix *A is not unique.*

2)If $\lambda_{1,} \lambda_{2,} \lambda_{3,\ldots\ldots,} \lambda_n$ be distinct eigen values of an *nxn* matrix then corresponding eigen vectors $X_{1,} X_{2,} X_{3,\ldots\ldots,} X_n$ form a linearly independent set.

3)If two or more eigen values are equal it may or may not be possible to get linearly independent eigen vectors corresponding to the equal roots.

4)Two eigen vectors X_1 and X_2 are called orthogonal vectors if $X'_1 X_2=0$

5)Eigen vectors of a symmetric corresponding to different eigen values are orthogonal.

Normalised form of vectors-

To find normalised form of $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, we divide each element by $\sqrt{a^2+b^2+c^2}$

For example, normalised form of $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ is $\begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$

$$\left[\because \sqrt{1^2+2^2+2^2}=3\right]$$