D. P. VIPRA COLLEGE BILASPUR

MATHEMATICAL METHOD
TOPIC : Properties of Laplace Transform

PROF. BHAGWAT KAUSHIK

Properties of Laplace Transform :

Linearity property : If a ,b ,c be any constants and f ,g ,h any function of **t,** then,

$$L[af(t) + bg(t) - eh(t)] = aL\{f(t)\} + bL\{f(t)\} - cL\{f(t)\}$$

$$LHS = \int_0^\infty e^{-st}\ \{af(t) + bf(t) - cf(t)\}dt$$

$$= a\int_0^\infty e^{-st} f(t)dt + b\int_0^\infty e^{-st} f(t)dt - c\int_0^\infty e^{-st} h(t)dt$$

$$= aL\{f(t)\} + bL\{f(t)\} - cL\{f(t)\}$$

The above Property of L is the reason for which L called a linear operator.

First shifting property : It states that the Laplace transform of the function $e^{at} f(t)$ is f(s-a) when f(a) is Laplace transform of f(t) and fix any real or complex number.

if ,

$L\{f(at)\} = \bar{f}(s)$ then

$L\{e^{at} f(t)\} = \bar{f}(s-a)$

By def ,

$$L\{e^{at} f(t)\} = \int_0^{\infty} e^{at} e^{-st} f(t)dt$$

$$= \int_0^{\infty} e^{-(s-a)t} f(t)dt$$

$$= \bar{f}(s-a)$$

Which proves the required condition. In order to demonstrate this method of Evaluation of Laplace transform.

Second Shifting Property : This property arise from the multiplication of the transform by an exponential function. At stage that if a function G(t) is defined as :

$$G(t) = \begin{cases} f(t-a) & ,t > a \\ 0 \ , & 0 < t < a \end{cases}$$

Then, $L\{f(at)\} = e^{-as} f(s)$

Where, f(s) is Laplace transform of F(t)

From def :

$$= L\{G(t)\} = \int_0^\infty e^{-st} G(t)dt$$

$$= \int_0^\infty e^{-st} G(t)dt + \int_0^\infty e^{-st} G(t)dt$$

$$= \int_0^\infty e^{-st} f(t-a)dt$$

$$= \int_0^\infty e^{-st} f(t-a)dt$$

$$= e^{-sa} \int_0^\infty e^{-sz} f(z)dz$$

$$= e^{-sa} f(s)$$

By substituting t-a = z In order to demonstrate this method of Evaluating the Laplace transform.

Change of scale Property : According to this property if the Laplace transform of the function $f(s)$ is $f(s)$ for s>a, then ,

$$L\{f(at)\} = {}^{1}/_{a}\, \bar{f}({}^{s}/_{a})$$

$$L\{f(at)\} = \int_0^{\infty} e^{-st} f(at)dt$$

$$= \int_0^{\infty} e^{-su/a} f(u) \frac{du}{a}$$

$$= \frac{1}{a} \int_0^{\infty} e^{-su/a} f(u)du$$

$$L\{f(at)\} = \frac{1}{a} \bar{f} \frac{s}{a}$$

THANK YOU !