EE 232: Lightwave Devices

Lecture #2 – Optical gain and laser cavities

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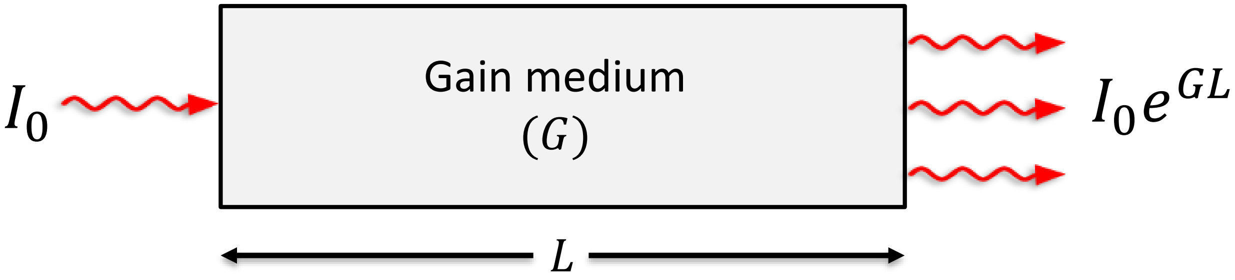
Gain Medium

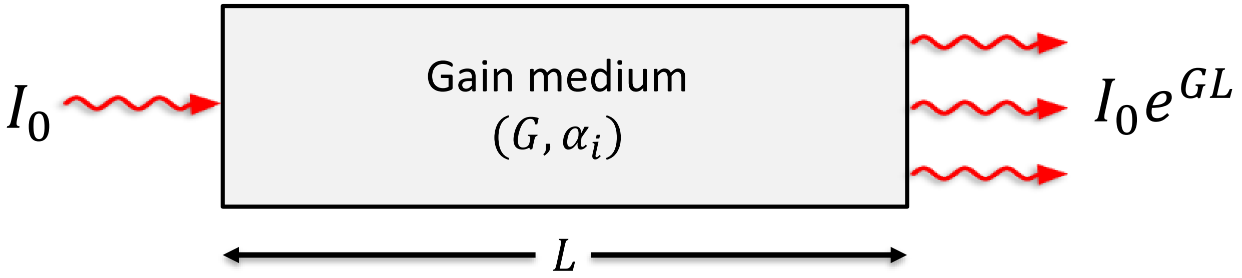


$$\begin{aligned}\Delta I &= I(z+\Delta L) - I(z) \\ &= I(z) + I(z)G\Delta L - I(z) \\ &= I(z)G\Delta L\end{aligned}$$

$$G = \frac{\Delta I}{I}\frac{1}{\Delta L}$$

Gain is the fractional increase
in light intensity per unit length
(Units are cm^{-1})

Gain Medium (with internal loss)

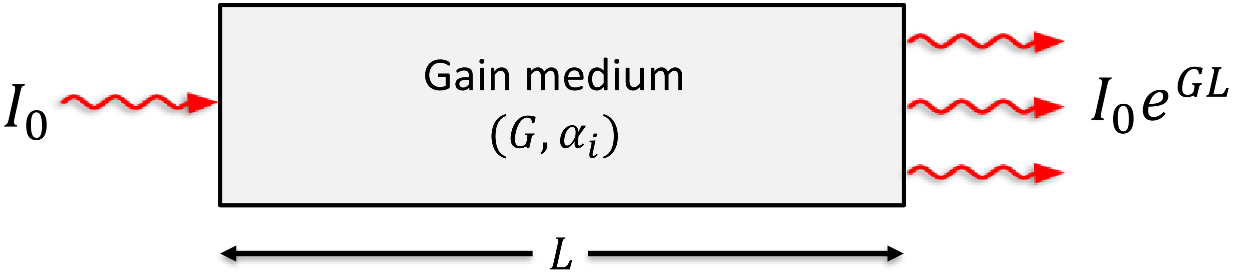


$$\begin{aligned}\Delta I &= I(z+\Delta L) - I(z) \\ &= I(z) + I(z)(G-\alpha_i)\Delta L - I(z) \\ &= I(z)(G-\alpha_i)\Delta L\end{aligned}$$

$$(G-\alpha_i) = \frac{\Delta I}{I}\frac{1}{\Delta L}$$

Internal loss (α_i) is the fractional decrease in light intensity per unit length (unrelated to fundamental absorption) (Units are cm^{-1})

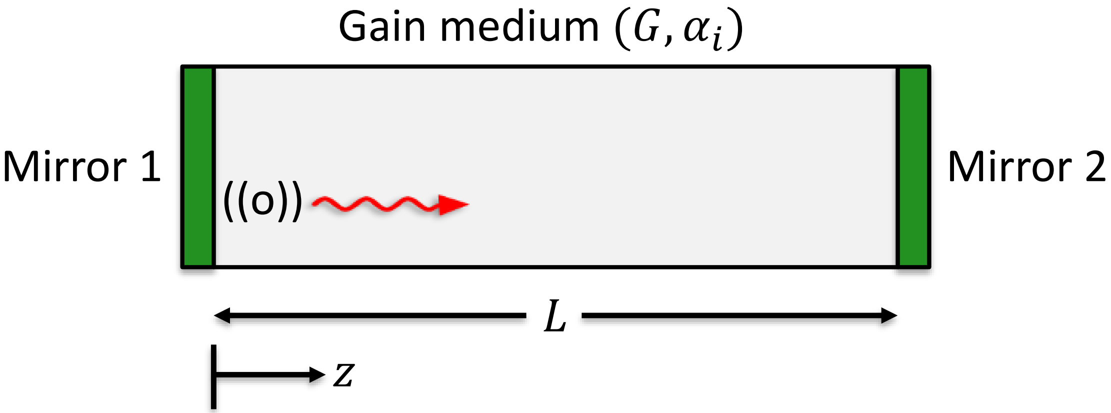
Gain Medium (with internal loss)



$$(G-\alpha_i) = \frac{\Delta I}{I}\frac{1}{\Delta L} \rightarrow \frac{dI}{dz}\frac{1}{I}$$

$$I(z) = I_0 e^{(G-\alpha_i)z}$$

Gain with cavity



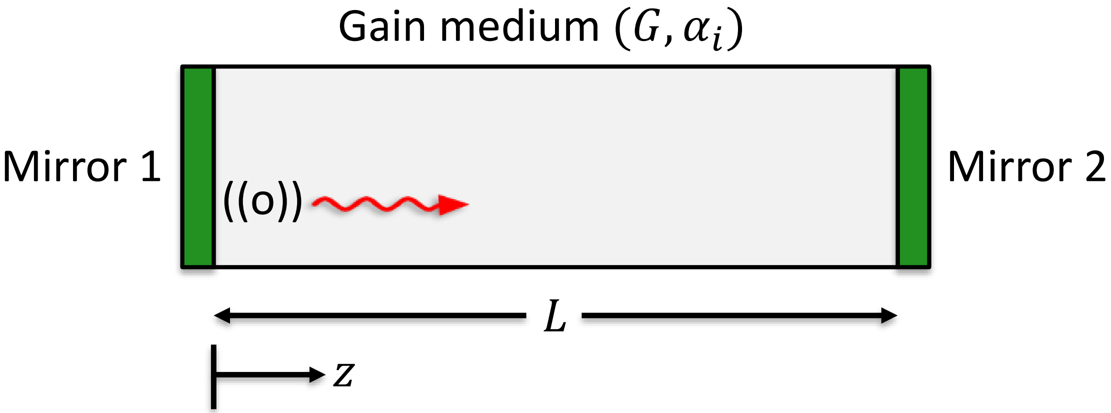
$$1 = \frac{I(z = 0^+ + 2L)}{I(z = 0^+)} = \mathrm{e}^{2(G_{th} - \alpha_i)L} R_2 R_1$$

(Threshold condition for self-sustaining oscillation)

↑

Round-trip gain

Gain with cavity

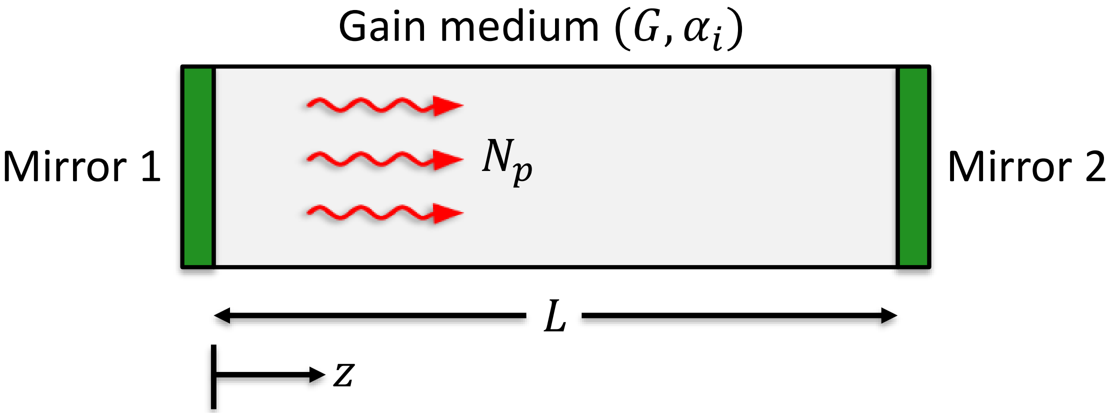


$$G_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_2R_1}\right) + \alpha_i$$

$$= \alpha_m + \alpha_i$$

(Threshold condition for self-sustaining oscillation)

Photon lifetime (τ_p)

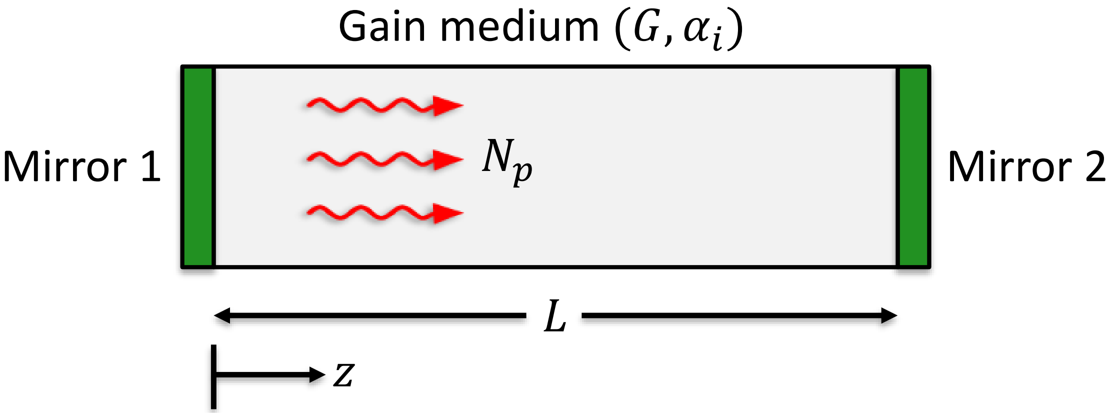


After one round trip: $(1-R_1R_2)N_p$ photons are lost from the cavity (ignoring α_i)

$$\frac{\Delta N_p}{\Delta t} = \frac{N_p(t+\tau_{RT}) - N_P(t)}{\tau_{RT}} = -\frac{(1-R_1R_2)N_p}{\tau_{RT}}$$

$$N_p = N_{p0}e^{-t/\tau_p}$$ where $$\boxed{\tau_p = \frac{2nL/c}{1-R_1R_2} \quad \text{(photon lifetime)}}$$

Quality factor (Q)

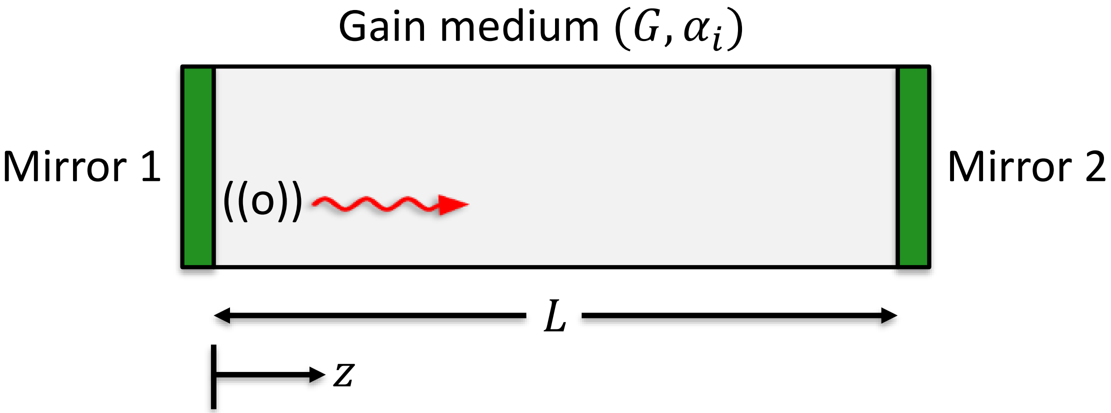


$$Q = 2\pi \frac{\text{Energy stored in cavity}}{\text{Energy lost per cycle}}$$

$$Q = 2\pi \frac{N_p \hbar\omega}{\frac{N_p \hbar\omega}{\tau_p} T} = \frac{2\pi}{T} \tau_p$$

$$\boxed{Q = \omega_0 \tau_p}$$

Alternative expression for G_{th}

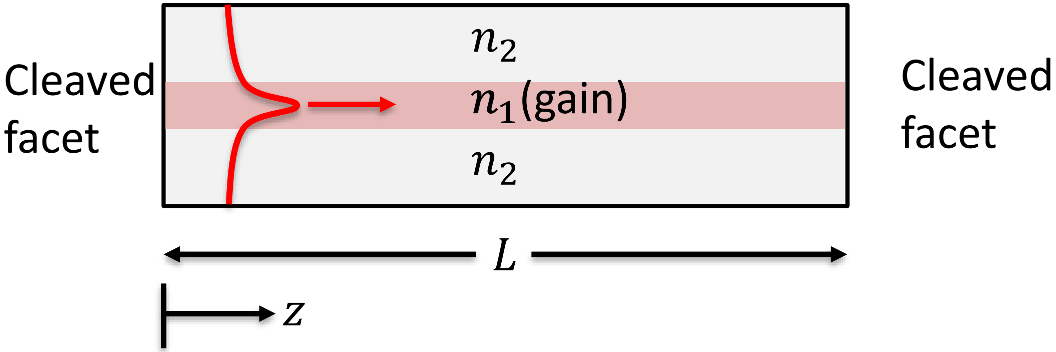


$G\dfrac{c}{n}$ is the fractional increase in photons per second

$$G_{th}\frac{c}{n}N_p = \frac{N_p}{\tau_p}$$

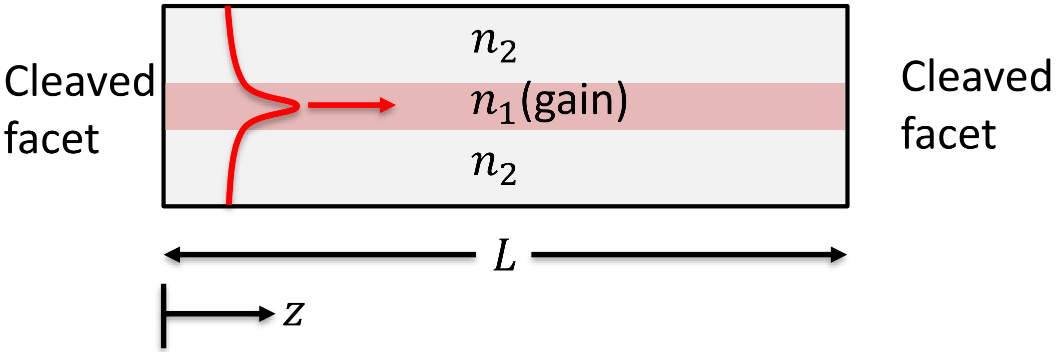
$$G_{th}\frac{c}{n}\tau_p = 1 \rightarrow \boxed{G_{th} = \frac{\omega_0}{Q}\frac{n}{c}}$$

Semiconductor laser



$$\boxed{R=\left(\frac{n-1}{n+1}\right)^2 \sim 30\% \quad \text{for } n=3.5}$$

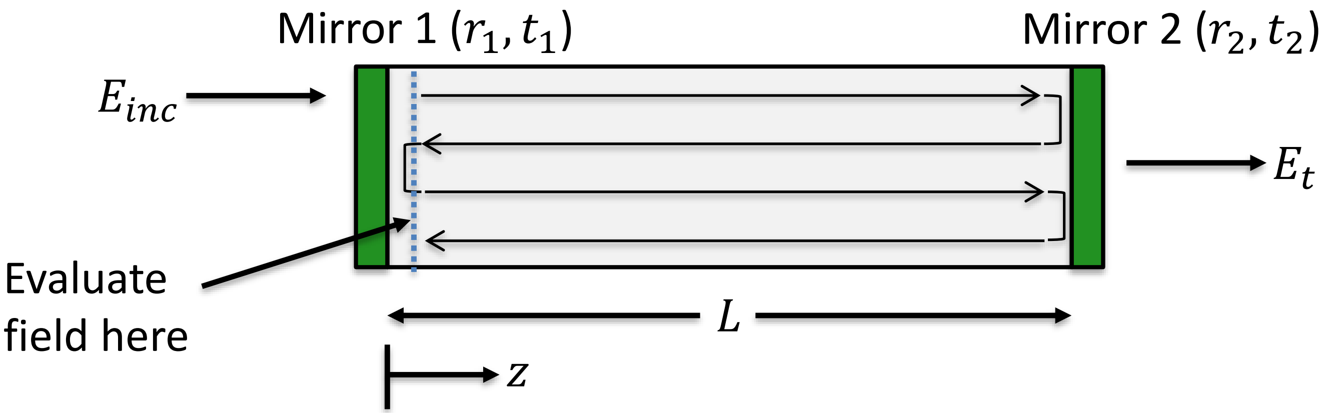
Semiconductor laser



$$\text{Confinement factor: } \Gamma = \frac{\text{Power of optical mode in gain region}}{\text{Total power of optical mode}}$$

$$\boxed{G_{th} = \frac{1}{\Gamma}\frac{\omega_0}{Q}\frac{n}{c}}$$

Fabry-Perot cavity modes



Field just to the right of Mirror 1 and propagating in +z direction:

$$E^+ = E_{inc}t_1(1 + r_1r_2e^{-jk2L} + r_1r_2r_1r_2e^{-jk4L} + (r_1r_2e^{-jk2L})^3 + \ldots)$$

$$\sum_k^\infty ar^k = \frac{a}{1-r}$$

$$E^+ = E_{inc}t_1\frac{1}{1-r_1r_2e^{-j2\theta}} \quad \text{where } \theta = kL$$

E^+ is the field traveling to the right (+z direction)

Fabry-Perot cavity modes

$$\text{Transmission } T = \frac{I_t}{I_{inc}} = \frac{\langle S_t \rangle}{\langle S_{inc} \rangle} = \frac{\dfrac{\left|E^+\right|^2 t_2^2}{2\eta_0}}{\dfrac{\left|E_{inc}\right|^2}{2\eta_0}}$$

Assume that $r_{1,2}$ and $t_{1,2}$ are real for simplicity

$$= \frac{\left|E_{inc} t_1 \dfrac{1}{1 - r_1 r_2 e^{-j2\theta}}\right|^2 t_2^2}{\left|E_{inc}\right|^2}$$

$$= \frac{t_1^2 t_2^2}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$$= \frac{(1 - r_1^{\ 2}) \eta_{cavity} \eta_0^{-1} (1 - r_2^{\ 2}) \eta_0 \eta_{cavity}^{-1}}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$$= \frac{(1 - r_1^{\ 2})(1 - r_2^{\ 2})}{\left|1 - r_1 r_2 e^{-j2\theta}\right|^2}$$

$\langle S \rangle$: Time-average Poynting vector (intensity)

η_0 : Impedance of medium outside cavity

η_{cavity} : Impedance of medium inside cavity

t_1: transmission of mirror 1

r_1: reflectivity of mirror 1

t_2: transmission of mirror 2

r_2: reflectivity of mirror 2

$$t_1^2 \eta_0 \eta_{cavity}^{-1} + r_1^{\ 2} = 1$$

$$t_2^2 \eta_0^{-1} \eta_{cavity} + r_2^2 = 1$$

Fabry-Perot cavity modes

$$= \frac{(1-r_1^2)(1-r_2^2)}{\left|1-r_1r_2e^{-j2\theta}\right|^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1r_2e^{j2\theta})(1-r_1r_2e^{-j2\theta})}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-r_1r_2\left[e^{j2\theta}+e^{-j2\theta}\right]+(r_1r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1r_2\left[\cos 2\theta\right]+(r_1r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1r_2\left[1-2\sin^2\theta\right]+(r_1r_2)^2}$$

$$= \frac{(1-r_1^2)(1-r_2^2)}{1-2r_1r_2+4r_1r_2\sin^2\theta+(r_1r_2)^2}$$

$$\boxed{T = \frac{(1-r_1^2)(1-r_2^2)}{(1-r_1r_2)^2+4r_1r_2\sin^2 kL}}$$

$\langle S \rangle$: Time-average Poynting vector (intensity)

η_0 : Impedance of medium outside cavity

η_{cavity} : Impedance of medium inside cavity

t_1: transmission of mirror 1

r_1: reflectivity of mirror 1

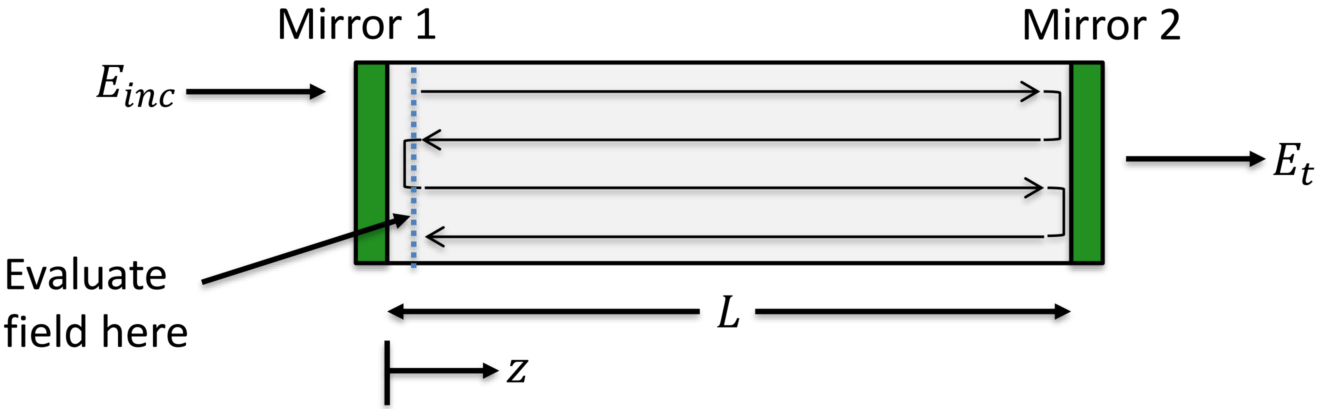
t_2: transmission of mirror 2

r_2: reflectivity of mirror 2

$$t_1^2\eta_0\eta_{cavity}^{-1} + r_1^2 = 1$$

$$t_2^2\eta_0^{-1}\eta_{cavity} + r_2^2 = 1$$

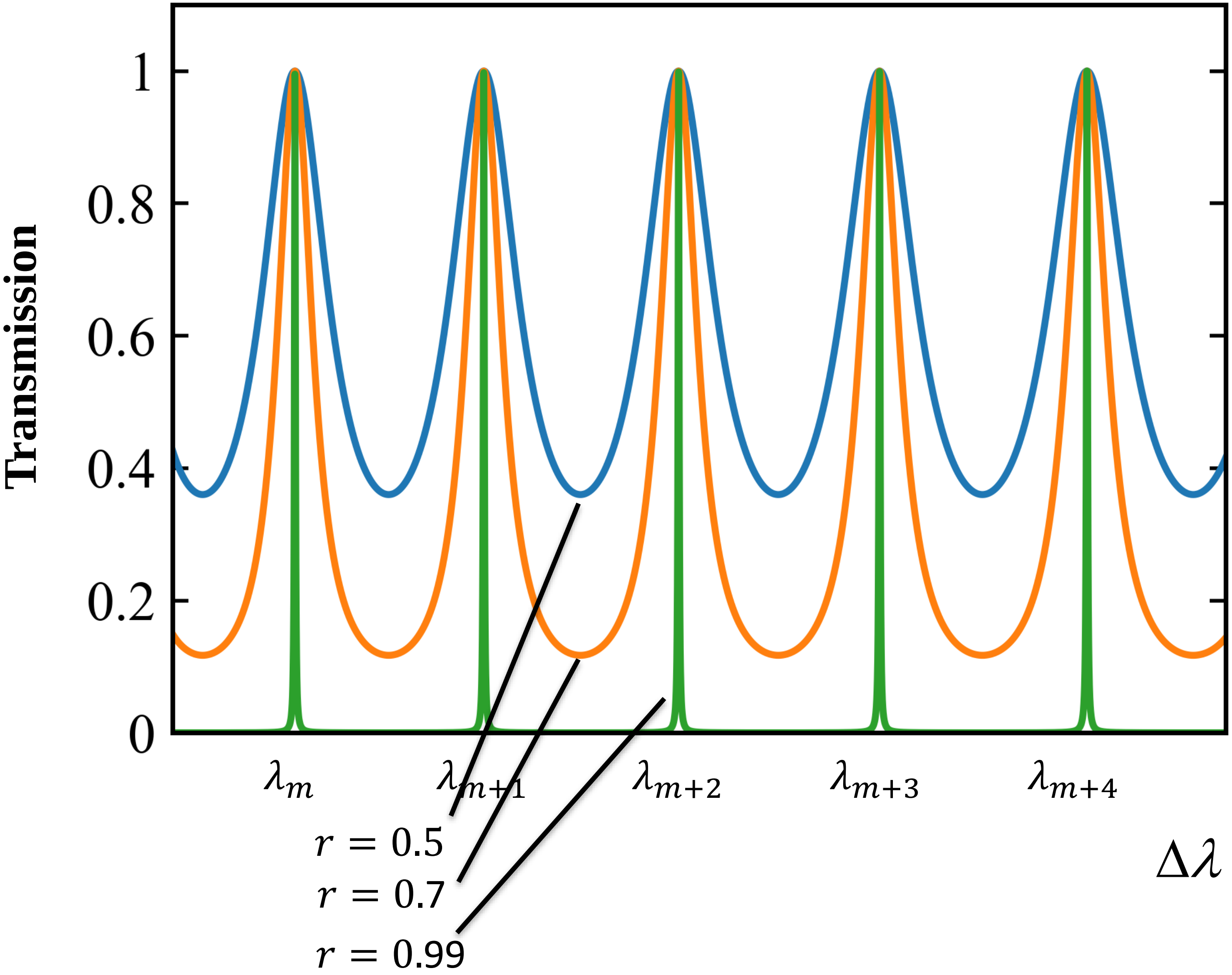
Fabry-Perot cavity modes



Resonance condition: $\sin^2\theta = 0 \rightarrow kL = m\pi$

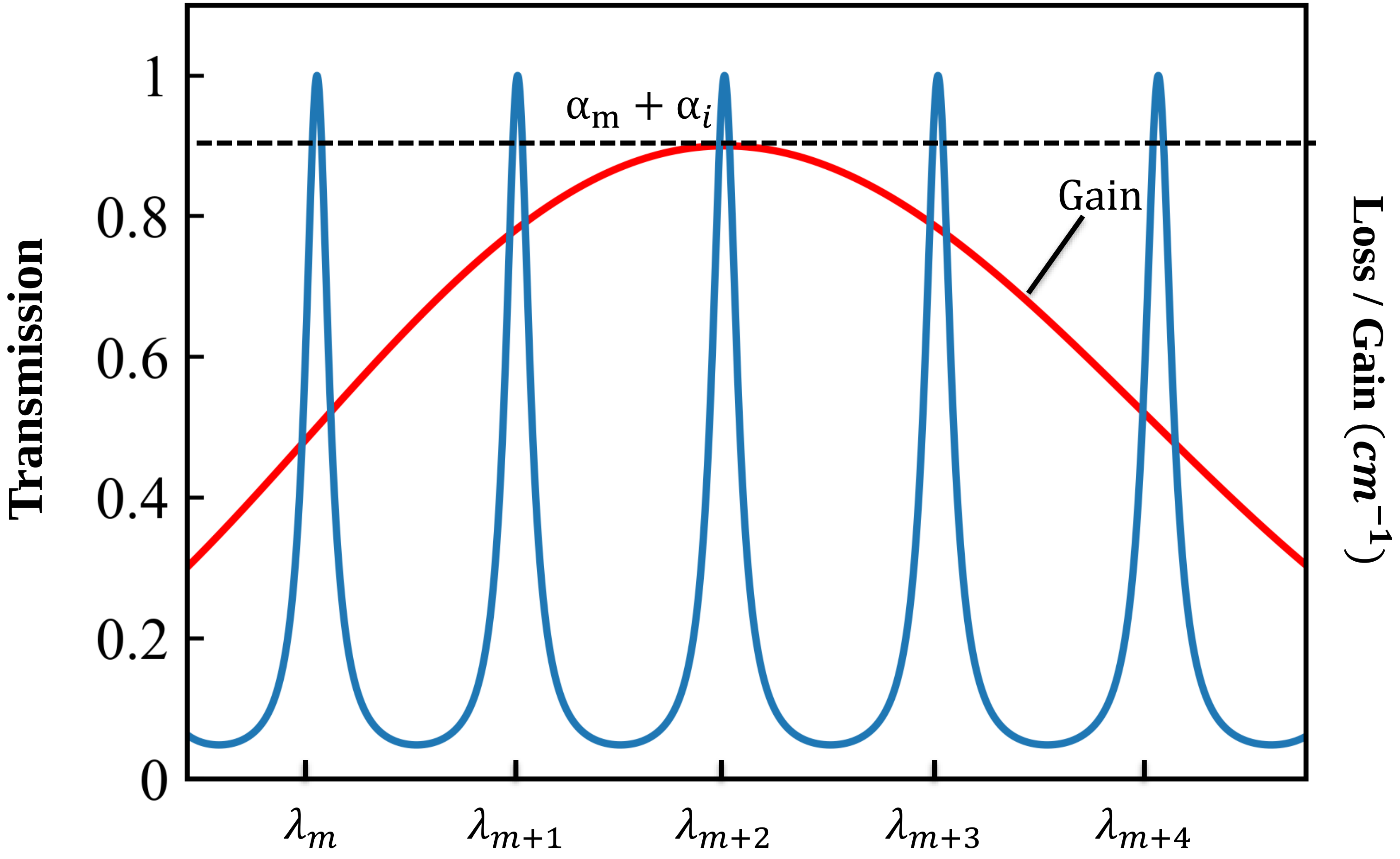
$$\boxed{L = \frac{m\lambda_0}{2n} \quad \text{m = integer}}$$

Fabry-Perot cavity modes



$$\Delta\lambda \cong \frac{\lambda^2}{2nL}$$

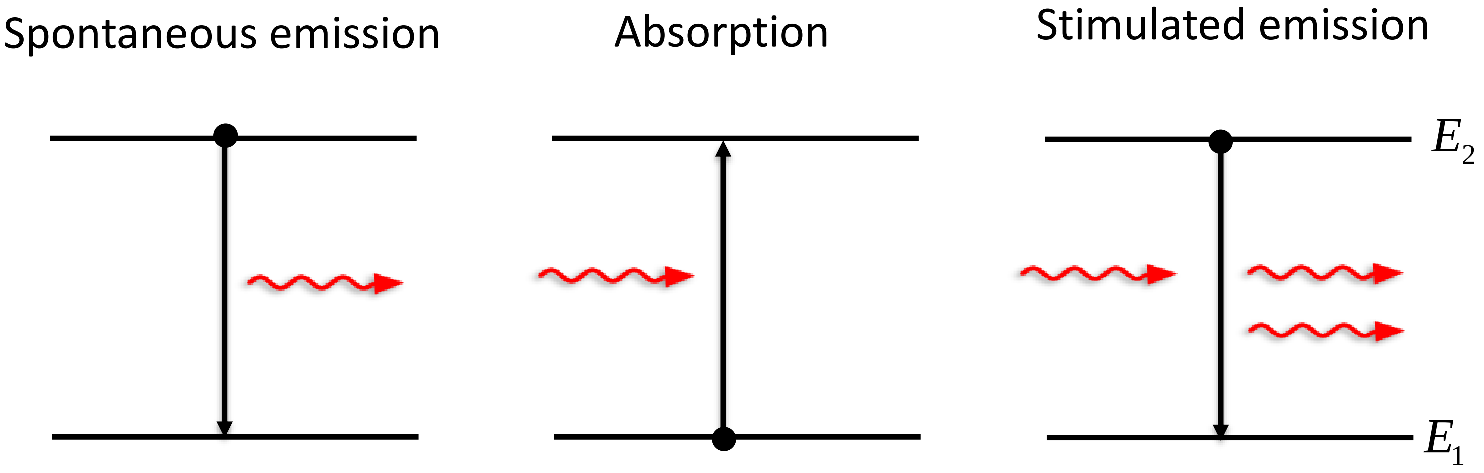
Fabry-Perot cavity modes



e.g. with simple gain curve

Threshold is achieved for one Fabry-Perot mode.

Population inversion



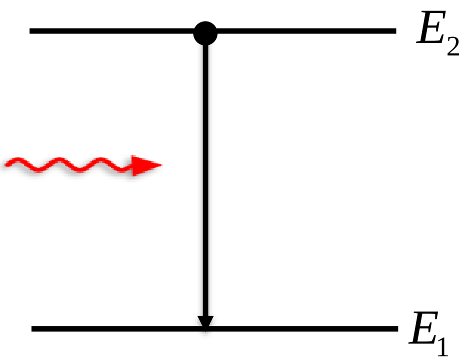
Gain will occur if stimulated emission > absorption which implies that number of electrons in the excited state is higher than the ground state (i.e. population inversion)

$$B_{21}WN_2 > B_{21}WN_1$$
$$N_2 > N_1$$

A_{21} = Spontaneous emission rate
$B_{21}W$ = Stimulated emission / absorption rate
W = Electromagnetic energy density
N_1 = Number of electrons in ground state
N_2 = Number of electrons in excited state
N_p = Number of photons

2-level system

$$\frac{dN_1}{dt} = -A_{21}N_2 + B_{21}WN_1 - B_{21}WN_2$$



At equilibrium,

$$0 = -A_{21}N_2 + B_{21}WN_1 - B_{21}WN_2$$

Let $N = N_1 + N_2$

$$N_1 = \frac{A_{21} + B_{21}W}{A_{21} + 2B_{21}W}N$$

$$N_2 = \frac{B_{21}W}{A_{21} + 2B_{21}W}N$$

For population inversion $N_2 > N_1$

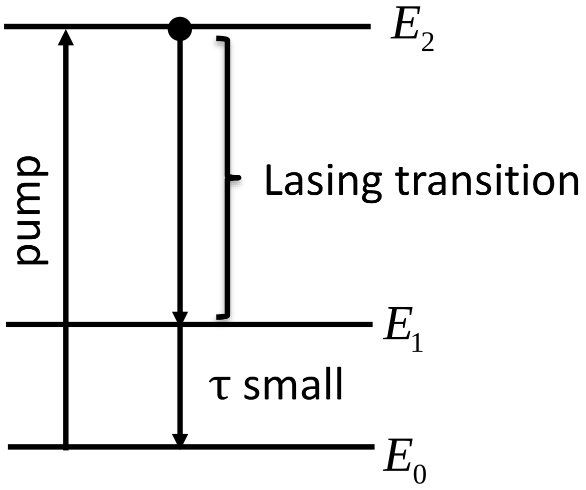
$$\frac{B_{21}W}{A_{21} + 2B_{21}W}N > \frac{A_{21} + B_{21}W}{A_{21} + 2B_{21}W}N$$

$$B_{21}W > A_{21} + B_{21}W \longleftarrow$$

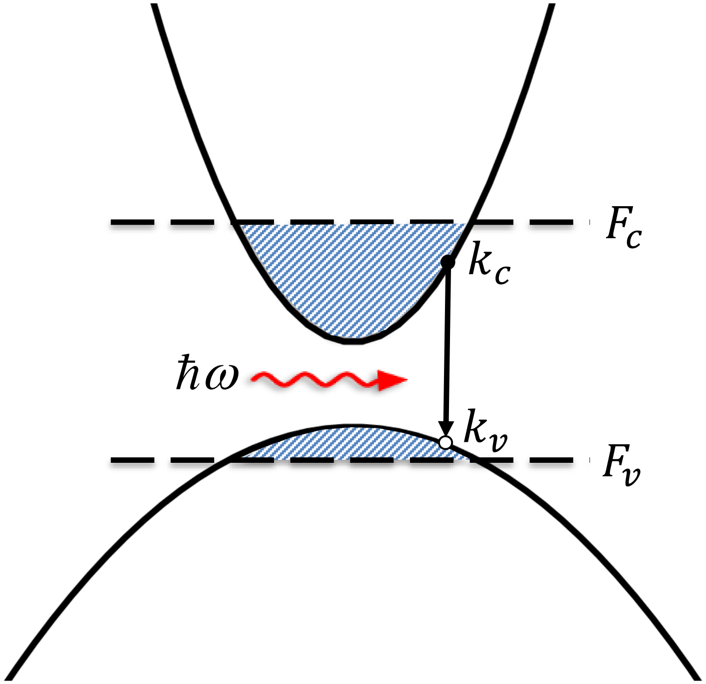
Not physical,
population inversion not possible
in a 2-level system (at equilibrium)!

3-level system

Population inversion ($N_2 > N_1$) is possible if a state with energy smaller than E_1 is added; provided electrons quickly decay from state 1 into state 0. In other words, A_{10} must be larger than A_{21}.



Gain in a semiconductor



$$f_c(k_c) = \frac{1}{1+\exp[(E(k_c)-F_c)/kT]}$$

$$f_v(k_v) = \frac{1}{1+\exp[(E(k_v)-F_v)/kT]}$$

$$\frac{dN_p}{dt} = B\rho f_c(1-f_v)N_p - B\rho f_v(1-f_c)N_p$$

Stimulated emission

Absorption

$$\frac{dN_p}{dt} > 0 \text{ for } f_c(1-f_v) > f_v(1-f_c)$$

$$\boxed{F_c - F_v > \hbar\omega}$$

Bernard-Duraffourg
Condition

$B =$ transition rate coefficient

$\rho =$ density of states

More on this as we progress in the course